Bethlehem Area School District

# Advanced Placement Calculus BC

# Curriculum



Julie Victory, Supervisor of Mathematics

Curriculum Committee

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**Name of Course: AP Calculus BC**

**Grade Level(s): 12**

**Unit 1: Improper Integrals**

**Estimated Instructional Time: 10 Days**

**PA Academic Standards:**

2.5.11 Mathematical Problem Solving and Communication

A. Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems
B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
C. Present mathematical procedures and results clearly, systematically, succinctly and correctly.
D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

2.11.11 Concepts of Calculus

A. Determine maximum and minimum values of a function over a specified interval.
B. Interpret maximum and minimum values in problem situations.
C. Graph and interpret rates of growth/decay.
D. Determine sums of finite sequences of numbers and infinite geometric series.
E. Estimate areas under curves using sequences and areas.

**New Standards Performance Standards:**

M1 Number and Operation Concepts
M5 Problem Solving and Mathematical Reasoning
M6 Mathematical Skills and Tools
M7 Mathematical Communication
M8 Putting Mathematics to Work

**Unit (Strand) Objectives:**

1-1 Use procedures for fitting integrands to basic integration rules
1-2 Recognize which basic integration rules to use in a given situation

**Enabling Objectives (Explore, Develop, Master):**

Find an antiderivative using integration by parts.
Use a tabular method to perform integration by parts.
Use trigonometric substitution to solve an integral.
Use integrals to model and solve real-life applications.
Understand the concept of a partial fraction decomposition.

Use partial fraction decomposition with linear factors to integrate rational functions.
Recognize limits that produce indeterminate forms.
Apply L'Hôpital's Rule to evaluate a limit.
Evaluate an improper integral that has an infinite limit of integration.
Evaluate an improper integral that has an infinite discontinuity.

**Text/Resource Materials:**
Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus of a Single Variable: Eighth Edition*. Boston: Houghton Mifflin, 2006.

Chapter 8

This course is explored through the interpretation of graphs and tables as well as analytic methods. The use of the graphing calculator is integrated throughout the course to provide a balanced approach to the teaching and learning of calculus that involves analytical, numerical, graphical, verbal, and written means. Students obtain solutions algebraically or analytically, support their results graphically or numerically with technology, and interpret results in the context of the problem. Students apply these methods to investigate and solve problems, to write about their conclusions, and to communicate orally with one another in small groups.

Each student uses a TI-83 or TI-89 graphing calculator daily in class activities, on homework, and on assessments when appropriate. Throughout the course, students demonstrate proficiency in producing the graph of a function in an arbitrary viewing window, finding the zeros of a function, computing the derivative of a function numerically, and computing definite integrals numerically. Emphasis is placed on the use of the graphing calculator to support, but not prove, results, and that analytic or algebraic techniques are necessary to prove results. Students are encouraged to use their graphing calculator to support their results when appropriate but are also presented many problem-solving experiences where the use of their graphing calculator is prohibited.

**Assessment:**
Quizzes
Take-home group projects including AP released exam items aligned with current topic of study
Homework
Classwork
Cumulative chapter tests in AP Exam format of half multiple-choice/half free-response

Students are expected to justify their answers in sentences on all forms of assessments and to be able to orally justify their work when called upon to do so.

**Extensions:**
Examine proving and explaining the proof of L'Hôpital's Rule.

**Remediation:**
Go to: http://www.mathgraphs.com/mg_calc8e.html . Work with these documents and

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| activities to focus on learning objectives for unit.<br><br>Review factoring techniques. |
| **Special Education** |
| **ESOL**<br>Use clear speech that avoids idioms and slang.<br>Do not require students to respond until they feel ready to speak before the group.<br>Employ some cooperative learning strategies.<br>Provide a focus on vocabulary.<br>Encourage students to bring to class an English/native language dictionary. |

**Name of Course: AP Calculus BC**

**Grade Level(s): 12**

**Unit 2: Infinite Series**

**Estimated Instructional Time: 17 Days**

**PA Academic Standards:**

2.5.11 Mathematical Problem Solving and Communication

A. Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems
B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
C. Present mathematical procedures and results clearly, systematically, succinctly and correctly.
D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

2.11.11 Concepts of Calculus

A. Determine maximum and minimum values of a function over a specified interval.
B. Interpret maximum and minimum values in problem situations.
C. Graph and interpret rates of growth/decay.
D. Determine sums of finite sequences of numbers and infinite geometric series.
E. Estimate areas under curves using sequences and areas.

**New Standards Performance Standards:**

M1 Number and Operation Concepts
M5 Problem Solving and Mathematical Reasoning
M6 Mathematical Skills and Tools
M7 Mathematical Communication
M8 Putting Mathematics to Work

**Unit (Strand) Objectives:**

2-1 Distinguish between sequence and series
2-2 Use appropriate test to determine whether a series converges or diverges
2-3 Form a Maclaurin or Taylor Series that is equivalent to a given function

**Enabling Objectives (Explore, Develop, Master):**

List the terms of a sequence.
Determine whether a sequence converges of diverges.
Write a formula for the nth term of a sequence.
Use properties of monotonic sequences and bounded sequences.

Understand the definition of a convergent infinite series.
Use properties of infinite geometric series.
Use the nth Term Test for Divergence of an infinite series.
Use the Integral Test to determine whether an infinite series converges or diverges.
Use properties of p-series and harmonic series.
Use the Direct Comparison Test to determine whether a series converges or diverges.
Use the Limit Comparison Test to determine whether a series converges or diverges.
Use the Alternating Series Test to determine whether an infinite series converges.
Use the Alternating Series Remainder Theorem to approximate the sum of an alternating series.
Classify a convergent series as either absolutely or conditionally convergent.
Rearrange an infinite series to obtain a different sum.
Use the Ratio Test to determine whether a series converges or diverges.
Use the Root Test to determine whether a series converges or diverges.
Find and compare polynomial approximations of an elementary function to the elementary function.
Find Taylor and Maclaurin polynomial approximations of elementary functions.
Use the remainder of a Taylor Polynomial.
Understand the definition of a power series.
Find the radius and interval of convergence of a power series.
Determine the endpoint convergence of a power series.
Differentiate and integrate a power series.
Find a geometric power series that represents a function.
Construct a power series using operations.
Find a Taylor or Maclaurin series for a function.
Find a binomial series.
Use a basic list of Taylor series to find other Taylor series.

**Text/Resource Materials:**
Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus of a Single Variable: Eighth Edition*. Boston: Houghton Mifflin, 2006.

Chapter 9

This course is explored through the interpretation of graphs and tables as well as analytic methods. The use of the graphing calculator is integrated throughout the course to provide a balanced approach to the teaching and learning of calculus that involves analytical, numerical, graphical, verbal, and written means. Students obtain solutions algebraically or analytically, support their results graphically or numerically with technology, and interpret results in the context of the problem. Students apply these methods to investigate and solve problems, to write about their conclusions, and to communicate orally with one another in small groups.

Each student uses a TI-83 or TI-89 graphing calculator daily in class activities, on homework, and on assessments when appropriate. Throughout the course, students demonstrate proficiency in producing the graph of a function in an arbitrary viewing

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| window, finding the zeros of a function, computing the derivative of a function numerically, and computing definite integrals numerically. Emphasis is placed on the use of the graphing calculator to support, but not prove, results, and that analytic or algebraic techniques are necessary to prove results. Students are encouraged to use their graphing calculator to support their results when appropriate but are also presented many problem-solving experiences where the use of their graphing calculator is prohibited. |
| **Assessment:**<br>Quizzes<br>Take-home group projects including AP released exam items aligned with current topic of study<br>Homework<br>Classwork<br>Cumulative chapter tests in AP Exam format of half multiple-choice/half free-response<br><br>Students are expected to justify their answers in sentences on all forms of assessments and to be able to orally justify their work when called upon to do so. |
| **Extensions:**<br>Explain and/or prove Taylor's Theorem (9.19)<br>Explain and/or prove Convergence of a Power Series (9.20) |
| **Remediation:**<br>Go to: http://www.mathgraphs.com/mg_calc8e.html . Work with these documents and activities to focus on learning objectives for unit. |
| **Special Education** |
| **ESOL**<br>Use clear speech that avoids idioms and slang.<br>Do not require students to respond until they feel ready to speak before the group.<br>Employ some cooperative learning strategies.<br>Provide a focus on vocabulary.<br>Encourage students to bring to class an English/native language dictionary. |

**Name of Course: AP Calculus BC**

**Grade Level(s): 12**

**Unit 3: Parametric Equations and Polar Coordinates**

**Estimated Instructional Time: 8 Days**

**PA Academic Standards:**

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C. Present mathematical procedures and results clearly, systematically, succinctly and correctly.
D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

2.11.11 Concepts of Calculus

A. Determine maximum and minimum values of a function over a specified interval.
B. Interpret maximum and minimum values in problem situations.
C. Graph and interpret rates of growth/decay.
D. Determine sums of finite sequences of numbers and infinite geometric series.
E. Estimate areas under curves using sequences and areas.

**New Standards Performance Standards:**

M1 Number and Operation Concepts
M5 Problem Solving and Mathematical Reasoning
M6 Mathematical Skills and Tools
M7 Mathematical Communication
M8 Putting Mathematics to Work

**Unit (Strand) Objectives:**

3-1 Use 3 variables to represent a curve in a plane
3-2 Describe graphs of parametric equations and confirm results analytically
3-3 Sketch a curve about a pole
3-4 Determine area in polar coordinates

**Enabling Objectives (Explore, Develop, Master):**

Sketch the graph of a curve given by a set of parametric equations.
Eliminate the parameter in a set of parametric equations.
Find a set of parametric equations to represent a curve.

Find the slope of a tangent line to a curve given by a set of parametric equations.
Find the arc length of a curve given by a set of parametric equations.
Find the area of a surface of revolution (parametric form).
Understand the polar coordinate system.
Rewrite rectangular coordinates and equations in polar form and vice versa.
Sketch the graph of an equation given in polar form.
Find the slope of a tangent line to a polar graph.
Find the area of a region bounded by a polar graph.
Find the points of intersection of two polar graphs.
Find the arc length of a polar graph.
Find the area of a surface of revolution (polar form).

**Text/Resource Materials:**
Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus of a Single Variable: Eighth Edition*. Boston: Houghton Mifflin, 2006.

Chapter 10

This course is explored through the interpretation of graphs and tables as well as analytic methods. The use of the graphing calculator is integrated throughout the course to provide a balanced approach to the teaching and learning of calculus that involves analytical, numerical, graphical, verbal, and written means. Students obtain solutions algebraically or analytically, support their results graphically or numerically with technology, and interpret results in the context of the problem. Students apply these methods to investigate and solve problems, to write about their conclusions, and to communicate orally with one another in small groups.

Each student uses a TI-83 or TI-89 graphing calculator daily in class activities, on homework, and on assessments when appropriate. Throughout the course, students demonstrate proficiency in producing the graph of a function in an arbitrary viewing window, finding the zeros of a function, computing the derivative of a function numerically, and computing definite integrals numerically. Emphasis is placed on the use of the graphing calculator to support, but not prove, results, and that analytic or algebraic techniques are necessary to prove results. Students are encouraged to use their graphing calculator to support their results when appropriate but are also presented many problem-solving experiences where the use of their graphing calculator is prohibited.

**Assessment:**
Quizzes
Take-home group projects including AP released exam items aligned with current topic of study
Homework
Classwork
Cumulative chapter tests in AP Exam format of half multiple-choice/half free-response

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| Students are expected to justify their answers in sentences on all forms of assessments and to be able to orally justify their work when called upon to do so. |
| **Extensions:**<br>Anamorphic pictures are drawings and paintings which appear distorted and almost unrecognizable to the unaided eye. Create anamorphic art by drawing a picture on a rectangular coordinate system then transferring it to a polar grid. Explain whether this is similar or not similar to a fun-house mirror which creates a distorted image of a normally proportioned person. |
| **Remediation:**<br>Go to: http://www.mathgraphs.com/mg_calc8e.html . Work with these documents and activities to focus on learning objectives for unit. |
| **Special Education** |
| **ESOL**<br>Use clear speech that avoids idioms and slang.<br>Do not require students to respond until they feel ready to speak before the group.<br>Employ some cooperative learning strategies.<br>Provide a focus on vocabulary.<br>Encourage students to bring to class an English/native language dictionary. |

**Name of Course: AP Calculus BC**

**Grade Level(s): 12**

**Unit 4: Calculus of Vectors**

**Estimated Instructional Time: 8 Days**

**PA Academic Standards:**

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B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
C. Present mathematical procedures and results clearly, systematically, succinctly and correctly.
D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

2.11.11 Concepts of Calculus

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B. Interpret maximum and minimum values in problem situations.
C. Graph and interpret rates of growth/decay.
D. Determine sums of finite sequences of numbers and infinite geometric series.
E. Estimate areas under curves using sequences and areas.

**New Standards Performance Standards:**

M1 Number and Operation Concepts
M5 Problem Solving and Mathematical Reasoning
M6 Mathematical Skills and Tools
M7 Mathematical Communication
M8 Putting Mathematics to Work

**Unit (Strand) Objectives:**

4-1 Examine vectors and vector-valued functions
4-2 Analyze the calculus of vector functions

**Enabling Objectives (Explore, Develop, Master):**

Express vectors in component form.
Perform arithmetic operations with vectors.
Find the angel measure between two vectors.
Use vectors to model and solve real-life applications.
Analyze planar curves in vector form.

Find velocity, acceleration, speed, and direction of motion.
Differentiate vectors.
Integrate vectors.
Determine height, flight time, and range for ideal projectile motion.
Analyze projectile motion with wind gusts.
Analyze projectile motion with air resistance.

**Text/Resource Materials:**
Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic: AP Edition*. Boston: Pearson Prentice Hall, 1999.

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**Assessment:**
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Homework
Classwork
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**Extensions:**
Computer graphics are typically created using vectors. Vector objects are graphically represented as arrows, with their tails at the origin, and a vector tip at some point in space. Vectors may be added, subtracted, scaled and rotated. Create an action figure

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| defined by many vertices and edges. Every vertex corresponds to a vector. The figure would move if a displacement vector were added to every vertex. The tip of the nose, toes, eyebrows, etc., would all move along some arrow, to a new place. Rotate the figure by translating it to the origin, rotating it, and translating it back to where it came from. |
| **Remediation:**<br>Go to: http://www.mathgraphs.com/mg_calc8e.html . Work with these documents and activities to focus on learning objectives for unit. |
| **Special Education** |
| **ESOL**<br>Use clear speech that avoids idioms and slang.<br>Do not require students to respond until they feel ready to speak before the group.<br>Employ some cooperative learning strategies.<br>Provide a focus on vocabulary.<br>Encourage students to bring to class an English/native language dictionary. |